Let the c be the number of cups originally in the barrel and w be the number of cups of wine in it.

For any given ratio of wine to volume equal to d, after a dilution there will be (c-3) × d total cups of such wine in the barrel. Thus, after the dilution there will equal d × (c-3)/c. So, to get the new dilution after a servant comes by, multiply the old dilution ratio by (c-3)/c.

In the original state the dilution was 1. So, after three servants come by, the dilution will be $((c-3)/c)^3$.

We know this will be equal to 1/2. Thus, we need to solve for c in the following equation:

$((c-3)/c)^3 = 1/2$

$(c-3)/c = 1/ 2^{1/3}$

$1 - 3/c = 1/ 2^{1/3}$

$3/c = 1 - (1/ 2^{1/3})$

$3/c = 1 - ((2^{1/3} -1)/ 2^{1/3})$

$c = 3 \times 2^{1/3} / (2^{1/3} -1) =\sim 14.54197$