Question: The Cash 5 game of the Texas Lottery requires the player to pick 5 numbers from 1 to 37. The game shall do the same thing with a random draw. At least two numbers must match for the player to win anything. What is the least number of tickets the player must buy to guarantee a win?

Answer: 22

Solution:

The first step to problems like this is to divide the ball pool into x group, where x is the number of balls drawn minus one. Since there are five balls in the draw, we divide 37 by 4 = 9.25. Each group must have an integer number of balls, so we will have three groups of 9 and one of 10. The idea being that at least one group will have two or more of the balls drawn by the lottery.

Our groups shall be:

Group 1 = 1 to 9

Group 2 = 10 to 18

Group 3 = 19 to 27

Group 4 = 28 to 37

The next question is how many tickets do we need to buy in group 1 so that at least one of them will match at least two of the balls drawn by the lottery? We may assume that at least two balls drawn by the lottery will be in the group of 9 numbers.

I contend the answer to that question is five tickets. I'll show you how to do it with five tickets, but I can't prove it can't be done with four. I hear there is a proof for this, but it's long and complicated.

There are combin(9,2) = 36 possible pairs of two numbers. We must cover all 45 of them with five tickets. Recall group 1 contains numbers 1 to 9. The four possible pairs are:

1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9
	2,3	2,4	2,5	2,6	2,7	2,8	2,9
		3,4	3,5	3,6	3,7	3,8	3,9
			4,5	4,6	4,7	4,8	4,9
				5,6	5,7	5,8	5,9
					6,7	6,8	6,9
						7,8	7,9
							8,9

Let's let our first ticket contain the numbers 1, 2, 3, 4, and 5. That will cover the 10 pairs where both numbers are five or less. That leaves us with:

1,6	1,7	1,8	1,9
2,6	2,7	2,8	2,9
3,6	3,7	3,8	3,9
4,6	4,7	4,8	4,9
5,6	5,7	5,8	5,9
	6,7	6,8	6,9
		7,8	7,9
			8,9

For my second ticket, I'm going to cover the numbers that appear most often in the pairs remaining. That would be a tie of 8 appearances each for 6 to 9. I'll arbitrarily pick 5 as the 5th number. So, my second ticket is 5-6-7-8-9. That will eliminate 10 more pairs, leaving us with:

1,6	1,7	1,8	1,9
2,6	2,7	2,8	2,9
3,6	3,7	3,8	3,9
4,6	4,7	4,8	4,9

For my third ticket, I'm going to do the same thing and remove the numbers that appear the most often. However, it's a tie again between every number except 5. So, I'll arbitrarily pick 1-2-3-8-9 for my third ticket. That will eliminate six pairs, leaving me with:

1,6	1,7		
2,6	2,7		
3,6	3,7		
4,6	4,7	4,8	4,9

For my fourth ticket, I'm definitely picking 6 and 7, since those numbers appear four times each. 1, 2, 3, 4, 8 and 9 appear once each. I'll arbitrary decide to pick 4, 8 and 9 for the other three. So, my fourth ticket is 4-6-7-8-9. That will leave me with:

1,6	1,7
2,6	2,7
3,6	3,7

My fifth ticket will be 1-2-3-6-7, which will cover all six of the remaining combinations.

I am aware there are many other ways to achieve the same goal with other tickets. However, if anyone can figure out a way to do it in four tickets, I'll be very impressed.

Use the same logic to buy five tickets for groups 2 and 3. For group 2, consisting of numbers 10 to 18, add 9 to every number in my five tickets above. Likewise, for group 3, add 18.

Group 4 has 10 numbers. There are combin(10,2) = 45 ways to pick two numbers out of 10, which are:

1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	1,10
	2,3	2,4	2,5	2,6	2,7	2,8	2,9	2,10
		3,4	3,5	3,6	3,7	3,8	3,9	3,10
			4,5	4,6	4,7	4,8	4,9	4,10
				5,6	5,7	5,8	5,9	5,10
					6,7	6,8	6,9	6,10
						7,8	7,9	7,10
							8,9	8,10
								9,10

If the first ticket for group 1 is 1-2-3-4-5, we are left with these two-number combinations:

1,6	1,7	1,8	1,9	1,10
2,6	2,7	2,8	2,9	2,10
3,6	3,7	3,8	3,9	3,10
4,6	4,7	4,8	4,9	4,10
5,6	5,7	5,8	5,9	5,10
	6,7	6,8	6,9	6,10
		7,8	7,9	7,10
			8,9	8,10
				9,10

If the second ticket is 6-7-8-9-10, we're left with:

1,6	1,7	1,8	1,9	1,10
2,6	2,7	2,8	2,9	2,10
3,6	3,7	3,8	3,9	3,10
4,6	4,7	4,8	4,9	4,10
5,6	5,7	5,8	5,9	5,10

If the third ticket is 1-3-5-7-9, we're left with:

1,6		1,8		1,10
2,6	2,7	2,8	2,9	2,10
3,6		3,8		3,10
4,6	4,7	4,8	4,9	4,10
5,6		5,8		5,10

If the fourth ticket is 2-4-6-8-10, we're left with:

1,6		1,8		1,10
	2,7		2,9	
3,6		3,8		3,10
	4,7		4,9	
5,6		5,8		5,10

If the fifth ticket is 1-3-6-8-10, we're left with:

	2,7		2,9	
	4,7		4,9	
5,6		5,8		5,10

If the sixth ticket is 2-4-7-9-x (where x = anything), we're left with:

5,6		5,8		5,10

If the seventh ticket is 5-6-10-x-y (where x and y are anything), we've covered all 45 combinations.

So, I have established a way to cover every pair in a set of 10 with seven tickets. I can't prove that it can't be done with six tickets, but I doubt it.

Remember that group 4 will cover numbers 28 to 37, so add 27 to each number in my tickets for group 4.

Thus, a way to guarantee a win is with these 22 tickets:

1: 1,2,3,4,5
2: 5,6,7,8,9
3: 1,2,3,8,9
4: 4,6,7,8,9
5: 1,2,3,6,8
6: 10,11,12,13,14
7: 14,15,16,17,18
8: 10,11,12,17,18
9: 13,15,16,17,18
10: 10,11,12,15,17
11: 19,20,21,22,23
12: 23,24,25,26,27
13: 19,20,21,26,27
14: 22,24,25,26,27
15: 19,20,21,24,26
16: 28,29,30,31,32
17: 33,34,35,36,37
18: 28,30,32,34,36
19: 29,31,33,35,37
20: 28,30,33,35,37
21: 29,31,34,36,x
22: 32,33,35,37,x

Where x = any number

I can't prove it can't be done with 21. If you fine a way, please let me know.